LARGE CARRIER AM
DSB-LC [Large Carrier] AM incorporates the carrier signal into the transmitted waveform. Because commercial broadcast stations use this method, it is commonly known as standard AM, or broadcast AM.

1. DSB-LC Model

2. DSB-_LC_ in Time Domain

\[ s_{am}(t) = m(t) \cos(2\pi f_c t) + c(t) \]

\[ s_{am}(t) = A_c m(t) \cos(2\pi f_c t) + A_c \cos(2\pi f_c t) \]

\[ s_{am}(t) = A_c \left[ m(t) + 1 \right] \cos(2\pi f_c t) \] EQ.1

EQ.1 suggests that DSB-LC can be created by adding a constant (dc level) to m(t) prior to performing DSB-SC modulation.

3. Eq. 1 also suggests that we can view the DSB-_LC_ signal as a carrier \( \cos(2\pi f_c t) \) with amplitude given by \( A_c [1+m(t)] \). We call this the carrier envelope

\[ a(t) = A_c[1 + m(t)] \]

For ease of detection it is critical that the envelope be always positive. The large carrier amplitude must be larger than the DSB-SC amplitude.

Mathematically,

\[ A_c [1 + m(t)] \geq 0 \quad \text{for all } t \]

Pictorially,

The simplest way to achieve this condition is to reduce the amplitude of the DSB-SC portion of the signal. The reducing factor is called the amplitude sensitivity \( ka \).
2. DSB-LC

DSB LC in Time Domain

with envelope

\[ A(t) = A_c [1 + \kappa_a m(t)] \]

where \( \kappa_a \) is chosen to ensure that

\[ \kappa_a \left| m(t) \right|_{\text{max}} \leq 1 \]

% MODULATION \( \equiv \kappa_a \left| m(t) \right|_{\text{max}} \times 100\% \)

Modulation Index \( \equiv \mu_a \equiv \kappa_a \left| m(t) \right|_{\text{max}} \)
Which is handy for finding 'u' from a graph of s(t)

\[
\mu = \frac{a(t)_{\text{max}} - a(t)_{\text{min}}}{a(t)_{\text{max}} + a(t)_{\text{min}}}
\]

Note:
1. \(0 \leq \mu\) always
2. \(\mu \leq 1\) for \(a(t)_{\text{min}} > 0\)

which is the desired condition for envelope detection.

\[
S_{\text{det}}(f) = \mathcal{F} \left[ \frac{a(t)}{b(t)} \right]
\]

\[
= \mathcal{F} \left[ \frac{\Re{a(t)} \cos(2\pi f t)}{b(t)} + \frac{\Im{a(t)} \sin(2\pi f t)}{b(t)} \right]
\]

\[
= \frac{A}{2} \left[ \mathcal{F} \left( S(f - f_c) \right) + \mathcal{F} \left( S(f + f_c) \right) \right]
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DSB-LC in Frequency Domain

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DSB-LC in Frequency Domain
• Single Tone Example
Consider the single test tone modulating signal

\[ m(t) = A_m \cos(2\pi f_m t) \]

Let the carrier signal be

\[ c(t) = A_c \cos(2\pi F_c t) \]

The AM signal will be

\[ s_{am}(t) = A_c \left[ 1 + \mu \cos(2\pi f_m t) \right] \cos(2\pi F_c t) \]

where: \( \mu = \frac{A_m}{A_c} \) = modulation index

The single tone AM spectrum is

\[ S_{am}(f) = \frac{1}{2} \frac{2A_c A_m}{f_c} \left\{ 1 + \mu \cos(2\pi f_m t) \right\} + \frac{1}{2} \frac{2A_c A_m}{f_c} \left[ \cos(2\pi F_c f_m) \right]^2 \]

The \( f_c \) of the 3rd carrier produces 3 delta functions; at \( \pm f_c \) - carrier

\[ \frac{1}{2} \left[ f_c \pm f_m \right] \text{ - offsets} \]

\[ \frac{1}{2} \left[ f_c \pm 2f_m \right] \text{ - sum} \]
Single Tone AM Power Spectral Density

$$PSD_{AM} = |S_{AM}(f)|^2 \left( \frac{\text{watts}}{\text{Hz}} \right)$$

From the PSD we know:

$$P_s = \sum \left| S_{am}(f) \right|^2 df$$

$$P_{linear} = \frac{A_m^4}{4} + \frac{A_m^4}{2} = \frac{A_m^4}{2}$$ for constant $f$

$$P_{base} = \frac{A_m^4}{16} + \frac{A_m^4}{16} = \frac{A_m^4}{8}$$

$$P_{side} = \frac{A_m^4}{16} + \frac{A_m^4}{16} = \frac{A_m^4}{8}$$

The most of the AM signal power is in the carriers, not in the message sidebands.
This DSB-SC is wasteful of transmit power, compared to that of DSB-SC, since at best DSB-SC puts 1/3 of the power into the message, whereas DSB-SC always puts 100% of the power into the message.

(a) \( S_{ds}(t) = 10 \cos(10^3\pi t) \)  
(b) \( S_{ds}(t) = 20 \cos(10^3\pi t) + 10 \cos(10^5\pi t) \cos(10^3\pi t) \)

\[ P_{eq} = \frac{20^2}{2} + \frac{10^2}{2} + \frac{20^2}{2} = 200 + 10.5 + 10.5 = 225 \text{W} \]
(b) $S_m(t) = 20 \cos(10^6 \pi t) + 10 \cos(10^6 \pi t) \cos(10^6 \pi t)$

$P_o = \frac{\omega_0}{\omega} + \frac{\omega_2}{\omega} = 200 + 12.5 = 212.5 \text{W}$

$P_c = 200 \text{W}, \quad P_s = 25 \text{W}$

Thus at 50% mismatch, only 12.5% of the transmitted power is contained in the mismatch.