Abstract-- This paper investigates the application of wavelet noise removal techniques to digital baseband signals. Non-linear filtering in the wavelet domain is used to improve the performance of the standard time domain correlation process for these signals. The proposed wavelet-based receiver computes the normalized cross correlation between the filtered wavelet coefficients of the received signal and wavelet coefficients that correspond to the known transmitted digital signals. Simulations are conducted comparing the wavelet receiver to the classical matched receiver.

I. INTRODUCTION

Digital baseband transmission systems are used in a variety of communication systems including Ethernet, telephone loops, and magnetic and optical storage. The characteristic of these systems have been studied extensively and are detailed in most introductory texts on communication systems. Two prevalent performance problems that must be combated by the system designer are line noise and intersymbol interference. The classical solution to these problems can be found in the use of pulse shaping and matched filter detection of the transmitted pulses. In contrast, this paper will investigate the use of wavelet decomposition and non-linear filtering techniques for the detection of baseband digital signals. To this end, a variation of the wavelet based de-noising techniques of [1] and [2] are applied to the problem of detecting digital signals transmitted through bandlimited channels in the presence of additive white Gaussian noise.

The concept of performing signal correlation in the wavelet domain, and its application to signals of many types can be found in the literature, for example [3, 4]. This paper is a continuation of the work of [3], to study the use of wavelet based correlation for digital signal detection.

The remainder of this paper is divided into the following sections; section II provides a tutorial on the generation and detection of digital baseband signals. Section III describes the discrete wavelet transform and the basics of wavelet filtering. Section IV discusses the wavelet domain receiver implementation, section V contains simulations and the results, and section VI contains a summary.

II. BASEBAND DIGITAL SIGNALS

The simplest of digital baseband communications systems consists of the transmission of binary data made up of pulses. In a Pulse Amplitude Modulated (PAM) system the data is mapped to signal waveforms \( s(t) \) of the form

\[
s(t) = \sum_{k=0}^{\infty} a_k g(t-kT) \tag{1}
\]

The quantity \( a_k \) is the signal amplitude and represents the sequence of transmitted information symbols from the source, \( g(t) \) is the pulse shape which may be selected to control the spectral characteristics, and \( T \) is the symbol duration. This process is shown in Fig 1. For the case of uncorrelated information symbols the power spectrum of the PAM signal \( S_s(f) \) can be shown to be [5]

\[
S_s(f) = \frac{\sigma_a^2}{T} \left| G(f) \right|^2 \tag{2}
\]

Where the power spectrum of the uncorrelated symbols is given by \( S_a(f) = \sigma_a^2 \), and \( G(f) \) is the spectrum of the pulse \( g(t) \).

The receiver process is shown in block diagram form in Fig 2. The transmitted signal is modified by the baseband channel with impulse response given by \( c(t) \), and additive white Gaussian noise \( n(t) \). The resulting received signal is given by

\[
r(t) = \sum_{k=0}^{\infty} a_k x(t-kT) + n(t) \tag{3}
\]
where \( x(t) = g(t) * c(t) \), and \(*\) indicates convolution [5]. At the receiver a matched filter with impulse response \( h(t) \) processes the received signal \( r(t) \) and then provides samples at the rate \( nT \) to the threshold detector. The input to the threshold detector is therefore

\[
 y(nT) = \sum_{k=-\infty}^{\infty} a_k p(nT-kT) + w(nT) \tag{4}
\]

where \( p(t) = x(t) * h(t) \), and \( w(t) \) is the filtered noise. Taking the coefficient \( p(0) = 1 \), Eqn. (4) may be written

\[
 y(nT) = a_k + \sum_{k=-\infty}^{\infty} a_k p(nT-kT) + w(nT). \tag{5}
\]

The first term on the right \( a_k \) represents the contribution of the \( n^{th} \) symbol. The additional terms in the summation represent interference created by the bandlimited channel caused by the smearing of the pulse wave shape. This effect is known as intersymbol interference (ISI). The presence of both ISI and noise introduce errors by the detector. Therefore system designers endeavor to create transmit and receive filters to reduce these effects.

In the absence of ISI the optimum receiver for the AWGN channel is the correlator or matched filter that is matched to the signal waveform \( s(t) \). The characteristics of this receiver are detailed extensively in the literature [5, 6, 7, 8]. The matched filter has an impulse response

\[
 h(t) = s(T-t), \quad 0 \leq t \leq T. \tag{6}
\]

which is a time reversed version of the transmitted signal \( s(t) \). The signal \( y(t) \) at the output of the matched filter can be computed via convolution. With no noise present the output of the filter at sample time \( T \) can be found to be [8]

\[
 y(T) = s(t) * h(t) = E_b. \tag{7}
\]

where \( E_b \) is the energy of the signal \( s(t) \). Assuming the received signal at the input to the matched filter is corrupted with Gaussian noise of zero mean and variance \( N_o / 2 \). The output of the matched filter will then be a random variable with a mean of \( E_b \) and variance \( E_b \cdot N_o / 2 \). The probability of bit error for antipodal signals can be shown to be [8]

\[
 P_e = Q\left( \sqrt{\frac{2 \cdot E_b}{N_o}} \right) \tag{8}
\]

where

\[
 Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{u^2}{2}} \, du. \tag{9}
\]

Eqn. (8), shows that the probability of error at the detector does not depend on the detailed signal and noise characteristics, but only upon the ratio of the signal energy to the noise energy per bit (SNR) [8]. Since the \( Q \) function decreases as its argument increases, for a given average signal energy, an increase in the noise level will result in a higher probability of transmission error.

In order to combat ISI caused by a bandlimited channel, the pulse shape is selected to ensure fast roll-off of the pulse frequency response. A popular choice is the raised cosine spectrum, which has \( \sin(x)/x \) time domain pulse shape given by [7]

\[
 g(t) = \frac{\sin(\pi t / T)}{\pi t / T} \cos(\alpha \pi t / T). \tag{10}
\]

The parameter \( \alpha, \quad 0 < \alpha < 1 \), is the roll-off factor. It controls the bandwidth of the resulting pulse. Fig 3 shows a raised cosine pulse in the time domain. The receive filter should be matched to the expected pulse shape \( g(t) \) of the received signal \( r(t) \).
III. WAVELET DOMAIN FILTERING

A. The Discrete Wavelet Transform

The discrete wavelet transform (DWT) of a sequence \( x(n) \) is given by [10],

\[
W(a,b) = \sum_{n} \frac{1}{\sqrt{a}} x(n) \psi^* \left( \frac{n-b}{a} \right),
\]

where \( a \) represents the scale factor and is usually of the form, \( a = a_0^J \) for \( J = 0,1,... \). The parameter \( b \) represents the translation in discrete time. The wavelet function \( \psi_{a,b}(n) = \psi((n-b)/a) \) is expanded in the time domain as \( J \) increases (and conversely contracted in the frequency domain). At each successive scale (octave) the bandwidth of the DWT output is half that of the previous scale, and thus can be sampled at half the rate in accordance with Nyquist’s rule. Therefore, the values of \( b \) can be restricted to \( b = K \cdot 2^J \) for \( K \) an integer. This represents decimation of the DWT output by a factor of two at each successive octave \( J \). These restrictions on the parameters \( a \) and \( b \), produce the decimated DWT which is orthogonal and time variant [10].

Decomposition of a set of \( N \) data points by the DWT results in a matrix of \( N \) coefficients that represent the data in the wavelet domain. This matrix contains all the information necessary to reconstruct the original signal input from the corresponding wavelet functions. The large coefficients represent good correlations of the input with the decomposing wavelet basis; conversely, the small coefficients represent poor correlations of the input with the decomposing basis function. For ease of notation, the wavelet coefficients can be identified with a continuous index and expressed as

\[
W(k) = [w_1, w_2, ..., w_N].
\]
IV. MATCHED FILTERING OF WAVELET COEFFICIENTS

Assuming proper sampling of the time domain received signal $r(t)$, the wavelet noise removal technique can be described by three steps: (1) transform data into the wavelet domain via the DWT, (2) apply a non-linear threshold to the DWT coefficients (to remove noise), and (3) perform the inverse wavelet transform on these coefficients, to produce the filtered waveform. The wavelet domain matched filter (WMF) does not accomplish the last step. Instead, the filtered (post threshold) DWT coefficients of the received signal are processed by the wavelet receive filter $h(k)$. The wavelet receive filter is designed as a matched filter constructed from the reverse ordered wavelet coefficients $w(k)$ of the prototype transmitted signal $s(t)$.

$$h(k) = w(N - k) \quad (14)$$

The filter output results in the correlation between the de-noised wavelet coefficients of the received signal $w'(k)$ and the pre-stored DWT coefficients of the prototype signals.

$$y(k = N) = w'(k) * h(k) \quad (15)$$

The advantage of the WMF results from the filtering of the noisy signals provided by the wavelet filters. Fig 6 shows a block diagram of the wavelet receiver using wavelet de-noising and wavelet received filter.

V. SIMULATIONS AND RESULTS

The classical time domain matched filter (TMF) receiver discussed in section II, is compared to the wavelet domain matched filter (WMF) receiver using computer simulations conducted using Matlab® software. The performance comparison was conducted via Monte Carlo simulations. Bit error rate curves were generated for the antipodal PAM signals and are depicted in Fig. 7. The WMF of Fig. 6 was implemented using the Symmlet 8 wavelet, hard thresholding, and a threshold value of $\sigma/2$. Additionally, only the first 64 (of the 256) wavelet coefficients were used in the correlation process. We found through experimentation that most of the energy of the PAM signals were retained in these coefficients.

Simulations for both the TMF receiver, and the wavelet receiver used the same noise scale. The pulse shaping filter $g(t)$ produced 256 samples per symbol. The PAM waveforms were generated using Eq. 10. In each simulation trial one of the two antipodal PAM signals were randomly selected and subjected to added white Gaussian noise (AWGN) to produce the simulated received signal. Numerous trials using different instances of AWGN were conducted at signal to noise ratios ranging from -6 dB to 10 dB. A sufficient number of trials were conducted to produce a smooth representative bit error rate curve. For comparison purposes, Fig. 8 and Fig. 9 display typical eye diagrams of the raised cosine pulse with and without the added noise.

The wavelet coefficients of the received signal were computed and the wavelet denoising technique of section III was applied. After the wavelet filtering, the first 64 (of the 256) surviving coefficients were then correlated with the corresponding coefficients of each of the prototype signals, using the wavelet receive filter of Eqn. (14). The correlation resulting in the largest product was selected as the correct transmitted signal.

The theoretical bit error rate (BER) was computed from Eq. 2 and is shown on Fig. 7 for comparison. Both the WMF and the TMF provided similar results, and both essentially achieved the theoretical value during the simulation. Although the performance of WMF did not surpass that of the classic TMF, it does provide an improvement in processing speed. This is a result of limiting the correlations to only 64 coefficients for the WMF vice the 256 used in the TMF.
VI. SUMMARY

This paper compares the performance of a wavelet-based receiver to that of the optimum detector for antipodal raised cosine pulses in the presence of AWGN. The receiver consists of computing the cross-correlation between discrete wavelet transform coefficients of the received noisy signal and that of the prototype (transmitted) signal. The procedure is enhanced by using standard wavelet noise removal procedures. The details of method were developed. Simulations of the performance of the proposed algorithm versus classical correlation methods were presented.

Future work in this area should consider performance of the method using noise sources other than AWGN, and additional baseband signal types such as multi-ton.

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VIII. REFERENCES


